VANILLA FOREX OPTIONS: GARMAN-KOHLHAGEN AND RISK REVERSAL/STRANGLE

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ABSTRACT. The pricing of vanilla Forex options using the Garman-Kohlhagen formula is described. The smile using the standard market quotes at-the-money, risk-reversal and strangle is also described.

1. INTRODUCTION

The note presents the pricing of vanilla Forex European options. The volatility data are described by the standard figures in the Forex market, with the smile represented by the at-the-money (ATM) volatility, risk reversals and strangles.

A Forex transaction is represented by an amount in foreign currency N_1 and amount in domestic currency N_2 (also called quote currency) and a payment date t_p, with N_1 and N_2 having opposite signs. The option on the forex transaction is represented by the underlying Forex transaction, a strike K, the expiry $\theta \leq t_p$ and a call/put feature. The strike is related to the amounts by $K = -N_2/N_1$.

There are two currencies involved and so two discounting curves are required. They are denoted P_D for the domestic currency and P_F for the foreign currency[1].

For a EURUSD transaction, buying EUR at a rate of 1.47 and for a nominal of 1,000,000 EUR, the market standard is to chose USD as the domestic currency and EUR as the foreign currency. The amounts are $N_1 = 1,000,000$ and $N_2 = -1,470,000$.

2. PRELIMINARIES

In this note, we assume that the exchange rate used is today's exchange rate (and not spot exchange rate, which generally is T+2 settlement).

The present value is computed from the Black formula with strike- and expiry-dependent volatility. The forward rate for (payment) date t and rate S_0 is given by

$$F_0^t = \frac{P_F(0,t)}{P_D(0,t)} S_0.$$

2.1. **Vanilla option.** The present value is computed in the domestic currency. The present value with the Black formula and a volatility σ is

$$\mathrm{PV}(S, \sigma) = N_1 P_D(0, t_p) \omega \left(F_0^{t_p} N(\omega d_+) - K N(\omega d_-) \right)$$

where $\omega = 1$ for a call, $\omega = -1$ for a put and

$$d_\pm = \frac{\ln\left(\frac{F_0^{t_p}}{K}\right) \pm \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}.$$

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[1]The P_F used here should not be confused with the P^F used in interest rate modelling, where F refers to forward curves.

The *relative delta* of the forward value with respect to the forward exchange rate is given by

(1) $$\Delta_F = \omega N(\omega d_+).$$

The *relative delta* of the present value with respect to the spot exchange rate is given by

$$\Delta_{\text{Relative}}(S) = \omega P_F(0,t)N(\omega d_+).$$

The *currency exposure* generated by the position is described in Section **??**.

The *relative gamma* (second order derivative with respect to the spot exchange rate) is given by

$$\Gamma_{\text{Relative}}(S) = P_F(0,t)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}{d_+}^2\right)\frac{1}{\sigma\sqrt{t}}\frac{1}{S}.$$

The *gamma* is the relative gamma multiplied by the appropriate notional

$$\Gamma(S) = N_1 \cdot \Gamma_{\text{Relative}}(S)$$

The *gamma spot* is the gamma multiplied by the spot rate

$$\Gamma_{\text{Spot}}(S) = S \cdot \Gamma(S)$$

The reason to compute this number is to be able to compute the (first order) impact of a relative change of spot rate (from S to $S(1+\epsilon)$) on the delta.

2.2. **Reverse quote.** The relative delta and gamma can also be calculated for *reverse quote*. By this we mean that the option is written in the non-standard currency order and we want the delta with respect to the standard quote. If we have a USD/EUR option (with EUR the domestic currency) priced in EUR, we may want to compute the spot sensitivity with respect to the market quote (EUR/USD). Let the standard quote Q be such that $Q = 1/S$. Let $f(Q) = \text{PV}(1/Q, \sigma)$.

The reverse quote relative delta is given by

$$\Delta_{\text{Relative,Reverse}}(Q) = f'(Q) = -\Delta_{\text{Relative}}(1/Q)\frac{1}{Q^2} = -\Delta_{\text{Relative}}(S)S^2.$$

The reverse quote relative gamma is given by

$$\Gamma_{\text{Relative,Reverse}}(Q) = f''(Q) = \left(\Gamma_{\text{Relative}}(S)S + 2\Delta_{\text{Relative}}(S)\right)S^3.$$

The *gamma* and *gamma spot* are also defined in the reverse quote case:

$$\Gamma_{\text{Reverse}} = N_1 \cdot \Gamma_{\text{Relative,Reverse}}$$

and

$$\Gamma_{\text{Spot,Reverse}} = Q \cdot \Gamma_{\text{Reverse}}.$$

3. SMILE

The smile is the description of the implied volatility for each strike, i.e. of a volatility function $\sigma = \sigma(K)$. A standard smile description is done with the at-the-money (ATM) volatility, risk reversals (RR) and smile strangles (SS) (see **?** for more details and other possibilities).

The delta used here is the delta forward described in (**??**). The *at-the-money* (ATM) used is the ATM - Delta Neutral Straddle (DNS), i.e. the strike for which the straddle has 0 delta forward. It is denoted K_{ATM}.

The risk reversals and strangles are given for some deltas (in general, for delta 0.10 and 0.25). It means that the figures given are related to puts with deltas -0.10 and -0.25 and to calls with deltas 0.25 and 0.10. All the options are out-of-the-money; the puts have strikes below the ATM strikes and calls have strikes above the ATM strike. The strikes are denoted $K_{x,T}$ with x the absolute value of the delta and T the type (C for call and P for put).

The risk reversals contain information about the skew (or slope) of the smile. The risk reversal figure is the difference between the volatility at the call and the volatility at the put:

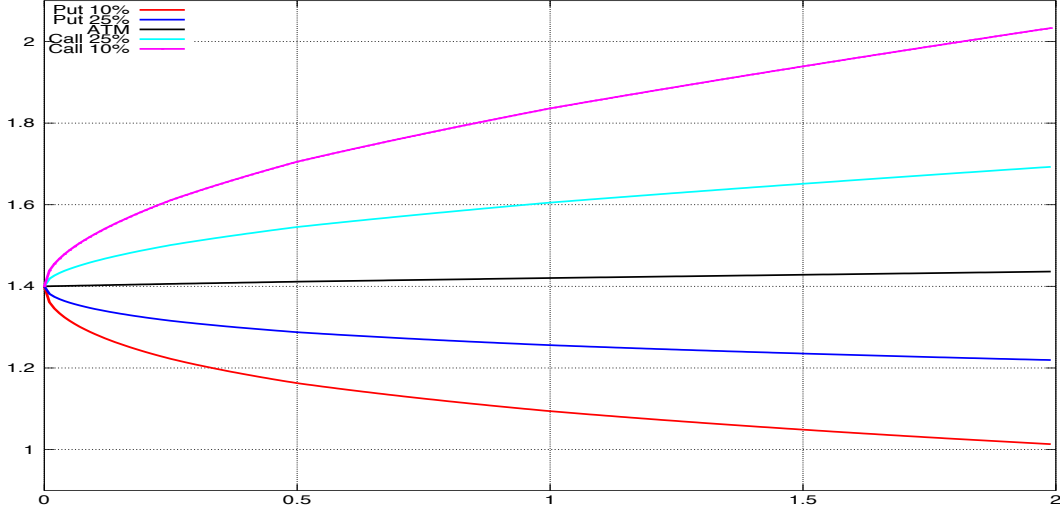$$\text{RR}(x) = \sigma(K_{x,C}) - \sigma(K_{x,P}).$$

FIGURE 1. Strikes for expiry up to 2 years and delta 10% and 25%.

The strangles contain information about the curvature of the smile. The strangle figure is the difference between the average of the volatility out-of-the-money and volatility at-the-money:

$$\mathrm{SS}(x) = \frac{1}{2}\left(\sigma(K_{x,C}) + \sigma(K_{x,P})\right) - \sigma(K_{\mathrm{ATM}}).$$

The strike can be computed explicitly from the forward delta by

$$(2) \qquad\qquad K = F\exp\left(-(\sigma\sqrt{t}\omega N^{-1}(\omega\Delta) - \frac{1}{2}\sigma^2 t)\right).$$

The smile is described with the figures

   (1) Deltas (usually 0.25 and 0.10).
   (2) ATM volatility.
   (3) Risk reversal for each delta.
   (4) Strangle for each delta.

The strikes/volatilities table is obtained by

   (1) Computing the wing volatilities from the risk reversals and strangles.
   (2) Computing the ATM strike from the ATM volatility.
   (3) Computing the put/call strikes for each delta from the wing volatilities with (**??**).

Figure **??** represents the strikes for delta 10% and 25% obtained from steps (1) and (2) above.

In the current implementation, volatility is interpolated linearly on strikes and extrapolated flat beyond extreme strikes.

4. CURRENCY EXPOSURE

For non-Forex instruments and Forex forward transactions, the currency risk is simply the present value in each currency.

This is obviously not the case for currency options, but is the amount in each currency required to neutralise the currency movements. It is dependent on the model used for the option valuation. By using the Black model, the change of volatility implied by the change of spot is not taken into account.

For vanilla options, the currency exposure for the foreign currency is

$$\Delta = N_1 \cdot \Delta_{\mathrm{Relative}}$$

and for the domestic currency it is

$$-\Delta_{\text{Relative}} \cdot N_1 \cdot \text{Spot} + \text{PV}.$$

In other words, if one buys an option at the fair price and at the same time sells $N_1 \cdot \Delta_{\text{Relative}}$ foreign currency units against receiving $N_1 \cdot \Delta_{\text{Relative}} \cdot \text{Spot}$ domestic currency units, there will be no currency position. The PV part of the currency exposure hedges the premium paid in the domestic currency.

5. Implementation

The class implementing the computation of the implied strike from a delta and a volatility is `BlackImpliedStrikeFromDeltaFormula`.

The smile data for one time to expiry can be stored in the class `SmileDeltaParameter`. It can be constructed from deltas, ATM, strangles and risk reversals as described above.

The term structure version of the data consisting of an array of the previous parameters and an array of times to expiry can be stored in the class `SmileDeltaTermStructureParameter`.

The pricing method for vanilla Forex option is `ForexOptionVanillaMethod`. There is a present value method and a currency exposure method.

Contents

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