OpenGamma Quantitative Research

Brazilian Swaps

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OpenGamma Quantitative Research n. 18 | First version: September 2013, this version: December 2013

Abstract

Description and pricing method for Brazilian swap is provided.

Contents

1 Introduction

In this note we will describe what is called a Brazilian swap. Technically this kind of swap is a Pre-DI swap. A Pre-DI Swap is a fixed-floating swap. Pre is the fixed rate and DI is the floating rate of the swap. The floating rate is the overnight interbank deposit average rate which is calculated exponentially on a 252-business-day basis. This floating rate is known as the CDI or overnight DI (Deposito Interbanco) rate. This rate is annualized and is calculated daily by the Centre for Custody and Financial Settlement of Securities (CETIP).
This swap has only one payment at maturity.
From now on we will use the notation $\delta = \frac{1}{252}$. This value of δ is specific to Brazilian swaps (and more generally other linear products in the Brazilian market such as deposits or bonds) but our note is valid for other values of δ.
This pricing methodology hereunder applies not only to overnight coupon but also to Ibor (Libor, Euribor...) with the same kind of coupon structure.
We will denote $(t_i)_{0\leq i\leq n}$ all the business days between the effective date t_0 and the maturity of the swap t_n.

2 Fixed leg

The pay off of the fixed leg is the notional accrued (using the fixed rate) on a daily basis from the effective date (or start date) of the swap to its maturity:

$$N \prod_{i=0}^{n-1} (1+k)^{\delta} = N \ (1+k)^{n \ \delta}$$

where N is the notional, k the fixed rate and n the number of business days between the effective date t_0 and the maturity of the swap t_n.
In the implementation, the value of the quantity $n \ \delta$ is calculated thanks to the BusinessDays/252 day count convention method.

3 Floating leg

The pay off of the floating leg is the notional accrued (using the floating rate) on a daily basis from the effective date (or start date) of the swap to its maturity:

$$N \prod_{i=0}^{n-1} (1+DI_i)^{\delta}$$

where DI_i is the floating overnight DI rate between t_i and t_{i+1}, consequently this rate is fixed at the $i+1$-th day from the effective date t_{i+1}.

4 Pricing

We assume the existence of a discount curve, we will denote $P^D(t,u)$ the discount factor between date t and u for $t \leq u$.
The pricing of the fixed leg is relatively easy and done through a classical discounting method.

For the pricing of the floating leg, we will use a forward curve. So we denote $DI(t,t_i,t_{i+1})$ the forward overnight DI rate between the two open days t_i and t_{i+1} as seen from the pricing date t. To be precise $DI(t,t_i,t_{i+1})$ is the forward rate corresponding to a coupon paying in t_{i+1} the notional accrued by $(1+DI_i)^{\delta}-1$.
Let $(\Omega,\mathcal{F},\mathbb{P})$ be a probability space. If we denote $\mathbb{E}^{i+1}$ the t_{i+1}-forward measure, the mathematical definition of the forward rate $DI(t,t_i,t_{i+1})$ is the following conditional expectation

$$(1+DI(t,t_i,t_{i+1}))^{\delta}-1=\mathbb{E}^{i+1}\left[(1+DI_i)^{\delta}-1|\mathcal{F}_t\right] \tag{1}$$

where $\mathcal{F}_t$ is the σ-algebra representing the information until time t (which is the known information at t). Now, to price the floating leg of a Brazilian swap, we will need an hypothesis concerning the relation between the two curves: the discount curve $u\mapsto P^D(t,u)$ and the forward curve $DI(t,t_i,t_{i+1})$. We define

$$\beta^{DI}(t,t_i,t_{i+1})=(1+DI(t,t_i,t_{i+1}))^{\delta}\,\frac{P^D(t,t_{i+1})}{P^D(t,t_i)}$$

And our hypothesis is to assume β^{DI} constant through time which means $\beta^{DI}(t,t_i,t_{i+1})=\beta^{DI}(0,t_i,t_{i+1})$ and we denote now this quantity $\beta_0^{DI}(t_i,t_{i+1})$.
This kind of hypothesis is relatively common, an example of its use for a slightly different kind of coupon can found in Quantitative Research (2012).
Therefore the price at time $t\le t_0$ of the floating leg is

$$\begin{aligned}
V_t &= N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-1}(1+DI_i)^{\delta}\,|\mathcal{F}_t\right]\\
&= N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-1}(1+DI(t_i,t_i,t_{i+1}))^{\delta}\,|\mathcal{F}_t\right]\\
&= N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-1}\beta_0^{DI}(t_i,t_{i+1})\frac{P^D(t_i,t_i)}{P^D(t_i,t_{i+1})}|\mathcal{F}_t\right]
\end{aligned}$$

Then because the quantity $\prod_{i=0}^{n-2}\left(\beta_0^{DI}(t_i,t_{i+1})\frac{P^D(t_i,t_i)}{P^D(t_i,t_{i+1})}\right)$ is known at time t_{n-2} and using the tower property, we have

$$V_t = N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-2}\left(\beta_0^{DI}(t_i,t_{i+1})\frac{P^D(t_i,t_i)}{P^D(t_i,t_{i+1})}\right)\beta_0^{DI}(t_{n-1},t_n)\mathbb{E}^n\left[\frac{P^D(t_{n-1},t_{n-1})}{P^D(t_{n-1},t_n)}|\mathcal{F}_{t_{n-2}}\right]|\mathcal{F}_t\right]$$

Let's notice that $t\mapsto\frac{P^D(t,t_{n-1})}{P^D(t,t_n)}$ is a martingale under the t_n-forward measure, so

$$V_t = N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-2}\left(\beta_0^{DI}(t_i,t_{i+1})\frac{P^D(t_i,t_i)}{P^D(t_i,t_{i+1})}\right)\beta_0^{DI}(t_{n-1},t_n)\frac{P^D(t_{n-2},t_{n-1})}{P^D(t_{n-2},t_n)}|\mathcal{F}_t\right]$$

Then doing some simplification

$$V_t = N\,P^D(t,t_n)\mathbb{E}^n\left[\prod_{i=0}^{n-3}\left(\beta_0^{DI}(t_i,t_{i+1})\frac{P^D(t_i,t_i)}{P^D(t_i,t_{i+1})}\right)\beta_0^{DI}(t_{n-2},t_{n-1})\beta_0^{DI}(t_{n-1},t_n)\frac{P^D(t_{n-2},t_{n-2})}{P^D(t_{n-2},t_n)}|\mathcal{F}_t\right]$$

Finally, if we repeat this procedure, we obtain the following formula

$$V_t = N\ P^D(t, t_n)\mathbb{E}^n \left[\prod_{i=0}^{n-1} \left(\beta_0^{DI}(t_i, t_{i+1})\right) \frac{P^D(t_0, t_0)}{P^D(t_0, t_n)} | \mathcal{F}_t \right]$$

As $\prod_{i=0}^{n-1} \left(\beta_0^{DI}(t_i, t_{i+1})\right)$ is deterministic and $t \mapsto \frac{P^D(t,t_0)}{P^D(t,t_n)}$ is a martingale under $\mathbb{E}^n$ (the t_n-forward measure), we have the following pricing formula:

$$\boxed{V_t = N\ P^D(t, t_n)\ \prod_{i=0}^{n-1} \left(1 + DI(t, t_i, t_{i+1})\right)^{\delta}}$$

Concerning the valuation of swaps that already start (at an effective date t_0 such as $t_0 < t$) we have to accrue the notional of both legs. In this way for the fixed leg

$$N_{accrued}^{fixed} = N\ \ (1 + k)^{m\ \delta}$$

And in this way for the floating leg

$$N_{accrued}^{floating} = N \prod_{i=0}^{m-1} (1 + DI_i)^{\delta}$$

Where $N_{accrued}^{fixed}$ (respectively $N_{accrued}^{floating}$) is the notional accrued of the fixed (respectively the floating) leg, m is the number of open days between the effective date (or start date) t_0 and the pricing date t, DI_i is the fixing of the overnight DI rate for the ith open day.
The implementation of greeks (eg. sensitivities to the discount and the forward curve) are computed using algorithmic differentiation in the OpenGamma Analytics library.

5 Curve construction using Brazilian swaps

In the previous paragraph, we were working with two different curves. If we are working in a one curve environment or if we assume that our Brazilian overnight swaps are used to build the discount curve, our formula still holds and simple arbitrage free argument (or assuming $\beta_0^{DI}(t_i, t_{i+1}) = 1$) gives us the following relation between the two curves

$$DI(t, t_i, t_{i+1}) = \left(\frac{P^D(t, t_i)}{P^D(t, t_{i+1})} \right)^{\frac{1}{\delta}} - 1$$

We are using this relation because we want to build a discount curve using Brazilian swaps, which means we are working in a one curve environment (ie the Brazilian overnight curve is used for discounting).

We are working in the multi curve framework of OpenGamma. To build a curve we firstly define a basket of instruments. In our case, we are using 12 Brazilian swaps with different maturities (1D, 2D, 3D, 3M, 1Y, 2Y, 3Y, 4Y, 5Y, 10Y, 15y, 20y).
Then we use a solver (a global root finding algorithm)to minimize the par spread which is the spread to be added to the market standard quote of the instrument for which the present value of the instrument is zero.

To accelerate the convergence, we also use the par spread sensitivity to the curve which is calculated using algorithmic differentiation.

Concerning the performance in the OpenGamma analytics library, building a discount curve using a basket with 12 Brazilian swaps takes approximately 0.03 seconds (to be precise 3197ms for doing it 100 times). These tests have been done using a 3.5 GHz Quad-Core Intel Xeon.

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